Joint CSIR-UGC NET for JRF and Eligibility for Lectureship

Section-A

General Aptitude

Volume-2

Contents

MATHEMATICAL AND NUMERICAL ABILITY

Numerical ability pertains to the understanding and application of numerical relations and dealing with numbers as symbols. A certain level of cognitive capacity is essential for dealing with numbers as symbols. This cognitive helps us to function at a definite level of mental abstraction. This is the reason why the psychologists call numerical ability as one aspect of 'Abstract Intelligence'. The questions for checking a person's level of numerical ability mostly deal with numerical comprehension, numerical retention, numerical reasoning, and numerical analysis.

1. NUMBER SYSTEM

Types of Numbers

Natural Numbers: All the counting numbers are called natural numbers. Thus N = {1, 2, 3,….}denotes the set of all natural numbers. In this sense all the positive integers are natural numbers. The number 1 is the least natural number and there are infinite natural numbers. The numbers –31, –5, 0, 3.68, $\frac{3}{5}$, are some examples of numbers who are not natural numbers.

On the basis of divisibility property of natural numbers there are two types of natural numbers– Prime and Composite.

Prime Numbers: A number greater than 1 is called a prime number if it is not perfectly divisible by any other number except 1 and the number itself. A prime numbers has only two factors. The lowest prime number is 2 which is the only even prime number and lowest odd prime number is 3. The Prime numbers between 1 to 100 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 87, 89, 97 which are 26 in total.

Composite Numbers: A natural number greater than 1, which is exactly divisible by at least one number in addition to 1 and the number itself is called a composite number. Thus a composite number has more than two factors. Examples of composite numbers are 4, 6, 8, 9, 10,…. A composite number has at least two factors other than one but there can be only one prime number. E.g. $4 = 2^2$ and $8 = 2^3$ etc.

When we factorize 36, we get $2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$. Where 2 and 3 are prime numbers.

Note: The number 1 is neither a prime number nor a composite number.

Whole numbers: The collection of numbers which consists of natural numbers with zero as member of that set is known as the set of whole numbers. W = {0, 1, 2, 3…} is a set of whole numbers are generated by using these digits.

Integers: The collection of numbers which consists of natural numbers, the negative of natural numbers and zero is called a set of integers. The set z = {… –3, –2, –1, 0, 1, 2, 3…} is a set of integers. The integers having no sign or (+) sign is called a positive integer and the integer having (–) sign is called a negative integer. –33 is a negative integer and +33 or 33 simply is a positive integer.

2. SIMPLE AND COMPOUND INTEREST

2.1 Simple Interest

If a person borrows some money from someone for a certain period then the borrower has to pay some extra money, called Interest (I) on the money borrowed for that period. The money borrowed is called Principal (P) and the total sum comprising of principal and the interest is called the Amount (A). If the interest on a certain sum borrowed for a certain period is reckoned uniformly, then it is called Simple Interest.

If amount = A, Principal = P, Interest = I, Time = T, Rate of Interest per annum = R.

(i) Amount = Principal + Interest or (P + I)

(ii) Simple Interest (I) $= \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}$ or $\frac{\text{PRT}}{100}$

(iii) Rate of Interest (R) $= \frac{\text{Simple Interest} \times 100}{\text{Principal} \times \text{Time}}$ or $\frac{\text{I} \times 100}{\text{P} \times \text{T}}$

(iv) Time (T) $= \frac{\text{Simple Interest} \times 100}{\text{Principal} \times \text{Rate}}$ or $\frac{\text{I} \times 100}{\text{P} \times \text{R}}$

(v) Principal (P) $= \frac{\text{Simple Interest} \times 100}{\text{Time} \times \text{Rate}}$ or $\frac{\text{I} \times 100}{\text{T} \times \text{R}}$

Example. A sum is lent at 10% p.a. Simple interest will get doubled in how many years?

Explanation: Sum will be doubled when simple interest (S. I.) = Principal (P). Therefore,

$$\text{Simple Interest = P} = \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}\text{, giving } T = 10 \text{ years.}$$

Example. A banker lent Rs. 6000 at 10% and Rs. 5000 at 12% at the same time and for same period of time. The banker received Rs. 2400 as total interest on both loans. For what period the banker had lent the amounts?

Explanation: If T = time for which amounts are loaned,

$$2400 = \frac{6000 \times 10 \times T}{100} + \frac{5000 \times 12 \times T}{100}\text{, giving T = 2 years.}$$

Example. Three persons separately borrow Rs. 51000 in all from a banker at 10% and returned with interest after 2, 5 and 6 years respectively. If the returned amounts are equal, what are the sums borrowed by each of them?

Explanation: $(\text{Sum})_1 = (\text{Sum})_2 = (\text{Sum})_3$. If P_1, P_2, P_3 be the sums borrowed.

$$P_1 + \frac{P_1 \times 10 \times 2}{100} = P_2 + \frac{P_2 \times 10 \times 5}{100} = P_3 + \frac{P_3 \times 10 \times 6}{100} \text{ giving}$$

$$\frac{6}{5}P_1 = \frac{3}{2}P_2 = \frac{8}{5}P_3 = K \text{ (say) giving.}$$

$$P_1 = \frac{5}{6}K, P_2 = \frac{2}{3}K, P_3 = \frac{5}{8}K$$

But $P_1 + P_2 + P_3 = 51000$, therefore

$$\frac{5}{6}K + \frac{2}{3}K + \frac{5}{8}K = 51000 \text{ or } \frac{20K + 16K + 15K}{24} = 51000$$

giving $K = 24000$. P_1, P_2 and P_3 can now be calculated.

2.2 Compound Interest

Money is said to be lent at Compound Interest when at the end of a year or other fixed period the interest that has become due is not paid to the lender, but is added to the sum lent, and amount thus obtained becomes the principal for the next period. The process is repeated until the amount of the last period and the final amount is the required compound interest.

(i) Compound Amount = Principal $(1+\text{Rate}/100)^{\text{Time}}$ = $P(1+R/100)^N$

(ii) Compound Interest (C.I.) = Compound Amount – Principal = A – P

= Principal $(1+\text{Rate}/100)^{\text{Time}}$ – Principal

(iii) Compound Interest $= P\left[\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\ldots-1\right]$ for rates of interest $R_1\%$, $R_2\%$ … for the successive years.

(iv) When T = N + F, where f = fraction of year. $C.I. = P\left[\left(1+\frac{R}{100}\right)^n\left(1+\frac{R\times f}{100}-1\right)\right]$

(v) When interest is payable over a fraction of year 'f' $C.L. = \left[P\left(1+\frac{R\times f}{100}\right)^{T/f}-1\right]$

(vi) $C.I. - S.I. = P\left[\left(\frac{R_1R_2 + R_2R_3 + R_3R_1}{(100)^2} + \frac{R_1R_2R_3}{(100)^3}\right)\right]$ for rate of interest $R_1\%$, $R_2\%$ and $R_3\%$ for successive years. If $R_3 = 0$, i.e. interest is to be calculated for 2 years.

$$C.I. - S.I. = P\frac{R_1R_2}{(100)^2}$$

Example. A man deposited a total sum of Rs. 88400 in the name of his two sons aged 19 and 17 years so that at the age of 21, both will get equal amounts. If the money is invested at the rate of 10% compound interest per annum what are the shares of his two sons?

Explanation: Let x and y be the shares of elder and younger sons respectively. The amount invested in their names fetch interest for 21 – 19 = 2 years and 21 – 17 = 4 years respectively. Since the two sons are to receive equal amounts when they attain 21 years. We have

$$x\left(1+\frac{10}{100}\right)^2 = y\left(1+\frac{10}{100}\right)^4$$

giving $$\frac{x}{y} = \left(1+\frac{10}{100}\right)^2$$

$$= \frac{121}{100}$$

Therefore $x : y$ = 121 : 100

$$x = 88400 \times \frac{121}{221} = Rs.48400$$

Example. What is the difference between S.I. and C.I. for 2 years on Rs.10,000/- when the rate of interest is 11% for the first year and 12% for 2nd year?

Explanation: For first year S.I. = C.I.

Therefore, the required difference = S.I. at 12% for one year and S.I. at 11% for 1 year on Rs. 10,000.

$$= \frac{12}{100} = \left(\frac{10000 \times 11}{100}\right) = \text{Rs. } 132$$

Rule: The difference between simple interest and compound interest for 2 years on Rs. S when the rate of interest is $R_1\%$ for the first year and $R_2\%$ for the second year is

$$C.I. - S.I. = \frac{S \times R_1 \times R_2}{10000}$$

If $R_1 = R_2$

$$C.I. - S.I. = S \times \frac{R^2}{10000}$$

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